Equivalent Circuit Modeling of Losses and Dispersion in Single and Coupled Lines for Microwave and Millimeter-Wave Integrated Circuits

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***Abstract*—Losses and dispersion in open inhomogeneous guided wave structures such as microstrips and other planar structures at microwave and millimeter-wave frequencies and in MMIC's have been modeled with circuits consisting of ideal lumped elements and lossless TEM lines. It is shown that, given a propagation structure for which numerical techniques to compute the propagation characteristics are available, an equivalent circuit whose terminal frequency and time-domain properties are the same as the structure can be synthesized. This is accomplished by equating the network functions of the given single or coupled line multiport with that of the model and extracting all the parameters of the equivalent circuit model by using standard parameter identification procedures. This equivalent circuit is valid over a desired frequency range and represents a circuit model which can be used to help design both analog and digital circuits consisting of these structures and other active and passive elements by utilizing standard CAD programs such as SPICE. In order to validate the accuracy and usefulness of the models, results for a mismatched 50-Ω line in alumina and a high-impedance MMIC line stub are included. In addition, for the case of coupled lines the results for a nominal 50-Ω, 10 dB coupler on alumina obtained by using the circuit model on SPICE are compared with rigorously computed values of the scattering parameters for the lossy dispersive system.**

I. Introduction

The ANALYSIS and design of circuits consisting of guided wave structures can be facilitated by the use of equivalent circuits. The propagation characteristics, that is, the propagation constant and the impedance of ideal closed waveguides can be modeled in terms of a transmission line or a distributed parameter circuit consisting of *LC* elements. Nonideal structures such as waveguides having lossy walls and slotted waveguides have been treated in terms of coupled transmission lines representing the orthogonal modes of the ideal structure [1]–[3]. For ideal structures the series impedance and shunt admittance per unit length of the equivalent distributed parameter structures are rational functions and hence are easily realized in terms of *LC* elements. This, however, is not the case for open structures such as microstrips and coplanar waveguides. Accurate evaluation of the propagation characteristics of these structures including dispersion, losses, and impedances requires numerical techniques (e.g., [4], [5]). The empirical equations representing CAD models that have been derived for the propagation parameters from these numerical results are in general irrational [6], [7] and do not lead to an equivalent circuit model with realizable series and shunt branches. These empirical models are therefore not very compatible with the standard circuit analysis and design techniques including the use of computer aided design tools such as SPICE for the analysis and design of circuits consisting in general of these lossy dispersive lines and other active, passive, linear, and nonlinear elements. Furthermore, reliable and accurate empirical models for many useful structures such as microstrips on a passivated substrate used in MMIC's and coplanar waveguides are not available.

Efforts have been made to model skin effect frequency-dependent losses in transmission lines for their time-domain characteristics by augmenting the lines by an *RL* network designed to simulate the skin effect losses. This technique has been used for the transient analysis of lossy interconnections in digital circuits and low-frequency power lines [8], [9]. The work on the equivalent circuit modeling compatible with computer-aided analysis and design of coupled lines has also been reported primarily for the case of lossless lines [10]–[13]. This approach is generalized here to include the effect of dispersion and losses at higher frequencies, and a systematic unified procedure to extract the equivalent circuit parameters for single as well as coupled dispersive lossy guided wave structures is presented. These equivalent circuits consist of ideal lumped linear elements and lossless TEM transmission lines. Therefore, standard CAD programs such as SPICE can be used to incorporate these as subcircuits in the analysis and design of high-frequency and high-speed circuits.

II. Single Dispersive Lossy Transmission Line Model

In order to derive the expressions for the series impedances, shunt admittances, and the parameters of the ideal line in the models, as shown in Fig. 1, the *ABCD* matrix elements of the dispersive lossy line two-port are equated

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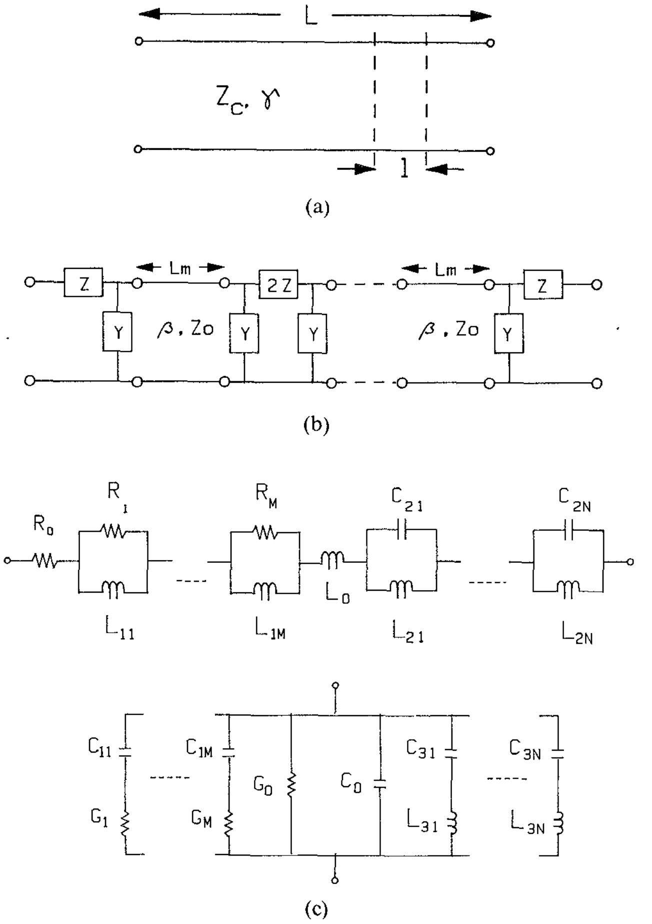


Fig. 1. (a) Schematic of general dispersive lossy line or waveguide. (b) The equivalent circuit model. (c) General form of lumped circuits for the series and shunt branches in (b).

with those of the model. That is,

$$\begin{bmatrix} \cosh\gamma l & Z_c \sinh\gamma l \\ \dfrac{\sinh\gamma l}{Z_c} & \cosh\gamma l \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\beta_0 L_m & jZ_0 \sin\beta_0 L_m \\ \dfrac{j\sin\beta_0 L_m}{Z_0} & \cos\beta_0 L_m \end{bmatrix} \cdot \begin{bmatrix} 1 & Z \\ Y & 1+ZY \end{bmatrix} \quad (1)$$

where Z_c is the characteristic impedance, γ is the complex propagation constant, and l is the length of the line to be modeled. Z_0, β_0, and L_m are the impedance, phase constant, and length of the ideal model line, and Z and Y are the impedances and admittances, respectively, of the two terminal lumped networks. Note that the line is divided into a number of sections with length l whose maximum value depends on the frequency range of validity of the model and is chosen so that Z and Y are realizable in a desired form. Equating the two independent matrix elements A and C (since $A = D$ and $AD - BC = 1$) gives

$$(1+2.ZY)\cos\beta_0 L_m + j\left[Z_0 Y(1+ZY) + \frac{Z}{Z_0}\right]\sin\beta_0 L_m = \cosh\gamma l \quad (2a)$$

$$2Y\cos\beta_0 L_m + j\left\{\frac{1}{Z_0} + Z_0 Y^2\right\}\sin\beta_0 L_m = \frac{\sinh\gamma l}{Z_c}. \quad (2b)$$

The expressions for the frequency-dependent complex series impedance and shunt admittance branch are derived from the above equations and are given by

$$Z = Z_c \frac{\cosh\gamma l - \sqrt{1 + j\sin(\beta_0 L_m)\cdot\sinh(\gamma l)\cdot Z_0/Z_c}}{\sinh(\gamma l)} \quad (3a)$$

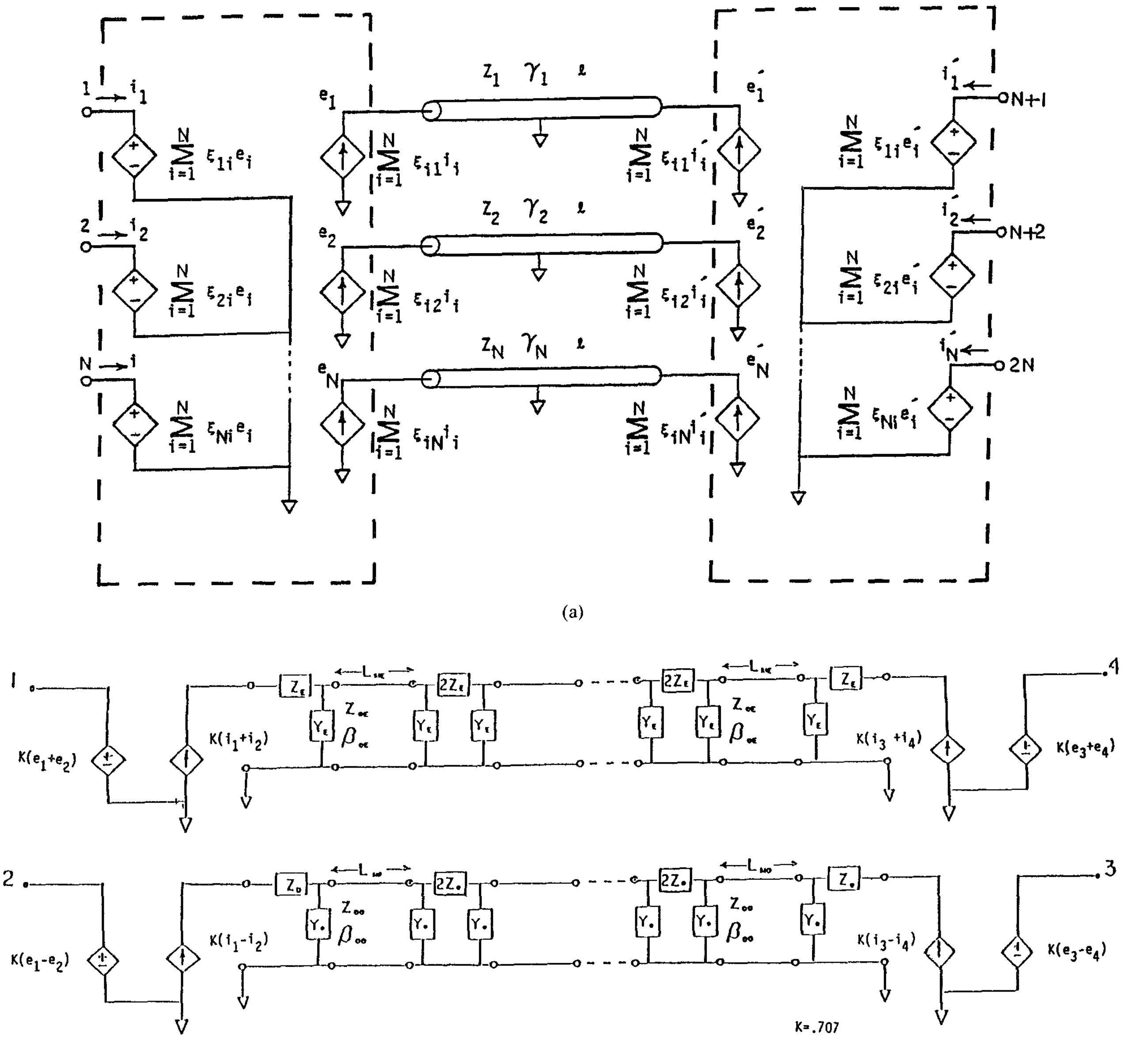
$$Y = \frac{j\cos(\beta_0 L_m) - j\sqrt{1 + j\sin(\beta_0 L_m)\cdot\sinh(\gamma l)\cdot Z_0/Z_c}}{Z_0 \sin(\beta_0 L_m)}. \quad (3b)$$

The model parameters to be determined include the length, the phase constant, and the impedance of the ideal TEM line in addition to the lumped circuits whose driving point impedance and admittance are given by (3a) and (3b). The lengths of the line to be modeled, l, and the model lossless TEM line, L_m, are not equal and are chosen such that Z and Y, the impedances and admittances to be synthesized, are realizable in a desired form with ideal RLC elements. The maximum value of the line length to be modeled, l, depends on the highest frequency at which the structure is to be modeled and the form of the augmenting lumped network. If we chose to realize Z and Y in the general modified Foster form shown in Fig. 1(c) with positive elements, then both functions must be positive real [14] over the desired frequency band for which the model is to be valid. We have synthesized these complex series and shunt branches in a modified Foster form, as shown in Fig. 1(c), by utilizing least-square minimization while optimizing the line length L_m in the model in an iterative manner.

The general form of impedance or admittance to be synthesized in the form shown in Fig. 1(c) is given by

$$Z \text{ or } Y = \sum_{i=0}^{N}\left[\frac{jA_i\omega}{(B_i^2 - \omega^2)} + \frac{j\omega D_i}{(C_i + j\omega)}\right]. \quad (4)$$

The constants in the above equation determine the element values in the model and can be found by using the least-square fit for the function while optimizing the pole locations by using the downhill simplex method and the line length L_m by parabolic interpolation. The error function representing the difference between the actual and the model values for the impedances and admittances over the frequency band is minimized in a least-square sense. As expected, the modeling accuracy is seen to improve when higher order networks are used for the series and shunt branches or when the lumped elements are allowed to acquire negative values.



(a)

(b)

Fig. 2. Multiple uniformly coupled dispersive lossy line model. All ξ's, Z's, and γ's are in general complex. (b) Equivalent circuit model for a pair of coupled lines.

III. Coupled Line Models

Coupled lossy dispersive lines are modeled as an interconnection of uncoupled lossy dispersive lines and mode coupling and decoupling networks. For lossless TEM and the quasi-TEM case these networks have been realized either in terms of congruent transformer banks [10] or linear dependent sources making the subcircuit mode compatible with CAD programs such as SPICE [12].

The equivalent circuit representing the n coupled line $2n$-port is derived from the normal mode solution of the multiple coupled line equations and the single line model developed in the previous section. The normal mode analysis of general lossy multiple coupled transmission lines is reasonably well known and is reviewed here in a concise simple form that is compatible with the straightforward computation of all the equivalent circuit model parameters. The voltage and currents in an n-line system are described by general transmission line equations:

$$-\frac{d}{dz}\begin{bmatrix}V]\\ I]\end{bmatrix} = \begin{bmatrix}0 & [Z_s]\\ [Y_{sh}] & 0\end{bmatrix}\cdot\begin{bmatrix}V]\\ I]\end{bmatrix} \tag{5}$$

where vectors $V] = [V_1, V_2, \cdots, V_n]^T$ and $I] = [I_1, I_2, \cdots, I_n]^T$ represent voltages and currents on the lines. T represents the transpose. $[Z_s] = [R] + j\omega[L]$; $[Y_{sh}] = [G] + j\omega[C]$, where ω is the frequency; and $[R]$, $[L]$, $[G]$, and $[C]$ are the series resistance, series inductance, shunt conductance, and shunt capacitance matrices whose elements represent the equivalent self and mutual parameters per unit length of the lines. The above system represents the generalized matrix eigenvalue and eigenvector problem and is readily solved for the state variables. Let $[M_v]$ be the complex voltage eigenvector matrix associated with the characteristic matrix $[Z_s][Y_{sh}]$. Then, following the same procedure as for the lossless case [12], it is shown that the voltage and current eigenvectors $e]$ and $j]$, respectively, are solutions of the decoupled set of equations

$$-\frac{d}{dz}\begin{bmatrix} e] \\ j] \end{bmatrix} = \begin{bmatrix} 0 & \operatorname{diag}[\gamma_k/y_k] \\ \operatorname{diag}[\gamma_k y_k] & 0 \end{bmatrix} \cdot \begin{bmatrix} e] \\ j] \end{bmatrix} \tag{6}$$

with

$$\begin{bmatrix} V] \\ I] \end{bmatrix} = \begin{bmatrix} [M_v] & [0] \\ [0] & \{[M_v]^T\}^{-1} \end{bmatrix} \cdot \begin{bmatrix} e] \\ j] \end{bmatrix} \tag{7}$$

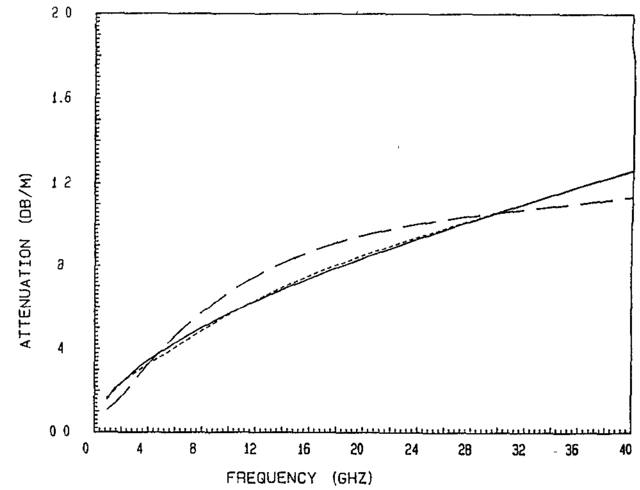
where γ_k is the propagation constant for the kth mode and is the square root of the kth eigenvalue of the $[Z_s][Y_{sh}]$ matrix and y_k is the characteristic admittance of the kth mode and is the corresponding element of the diagonal matrix $[Y_k]$ as given by

$$[Y_k] = [M_v]^{-1}[Y_{sh}]]M_v]. \tag{8}$$

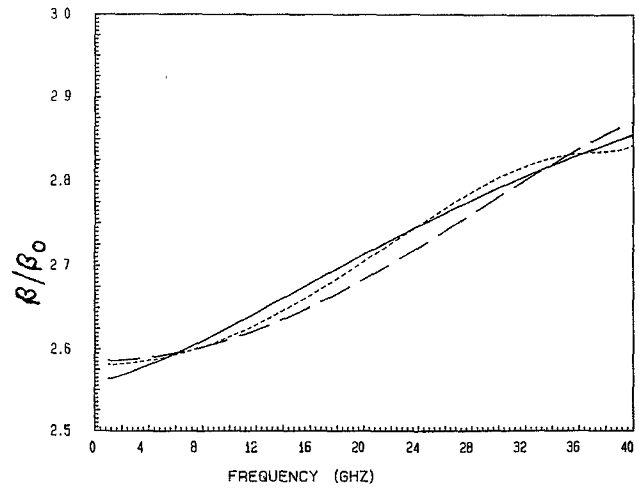
The above equations lead to the circuit model representing the coupled lines as shown in Fig. 2(a). The model consists of lossy uncoupled lines and a modal decoupling network at the input and a complementary coupling network at the output end. This network consists of linear real or complex dependent sources whose values ξ_{ij} are given by the elements of the voltage eigenvector matrix $[M_v]$. The main difference between this model and the one for the lossless case is that here the uncoupled lines are dispersive and lossy, having frequency-dependent complex impedances and propagation constants. In addition, the dependent sources are not generally in phase with the independent variables. The uncoupled lossy lines in the model represent the normal modes of propagation, and given the frequency-dependent real and imaginary parts of the propagation constant and impedances of these lossy lines, they are modeled as a two-port consisting of lossless lines and lumped elements, as shown in the previous section. In addition, ξ_{ij}, the elements of $[M_v]$, are real for many useful cases such as coupled identical lines or near-degenerate lines where the eigenvalues are close to each other. The general equivalent circuit representing a pair of coupled lossy dispersive lines is shown in Fig. 2(b).

IV. Examples

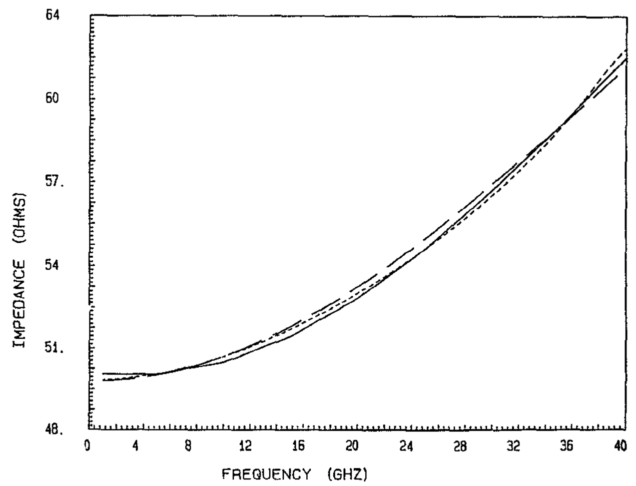
In order to validate the accuracy of the modeling technique and demonstrate the usefulness of the circuit models, results for typical single and coupled line circuits obtained by using the model are compared with those computed directly by using the complex equations for the single and coupled line immittance and scattering parameters. For the case of single lines, a 50-Ω microstrip on alumina and a high-impedance MMIC line on a passivated SI GaAs substrate are considered here to exemplify the effects of dispersion and losses at higher frequencies. For the case of coupled lines, a 50-Ω edge-coupled microstrip four-port on alumina with nominal 10-dB coupling is considered. The frequency-dependent propagation constants including losses for these structures were computed by using the spectral-domain techniques and reliable



(a)



(b)



(c)

Fig. 3. Propagation characteristics of the equivalent circuit model with first- and third-operator augmenting lumped element circuits (Z and Y). — Computed values for the line; --- values calculated for the model with the first order augmenting network; ··· values calculated with a third-order augmenting network. (a) Attenuation constant. (b) Normalized phase constant. (c) Characteristic impedance.

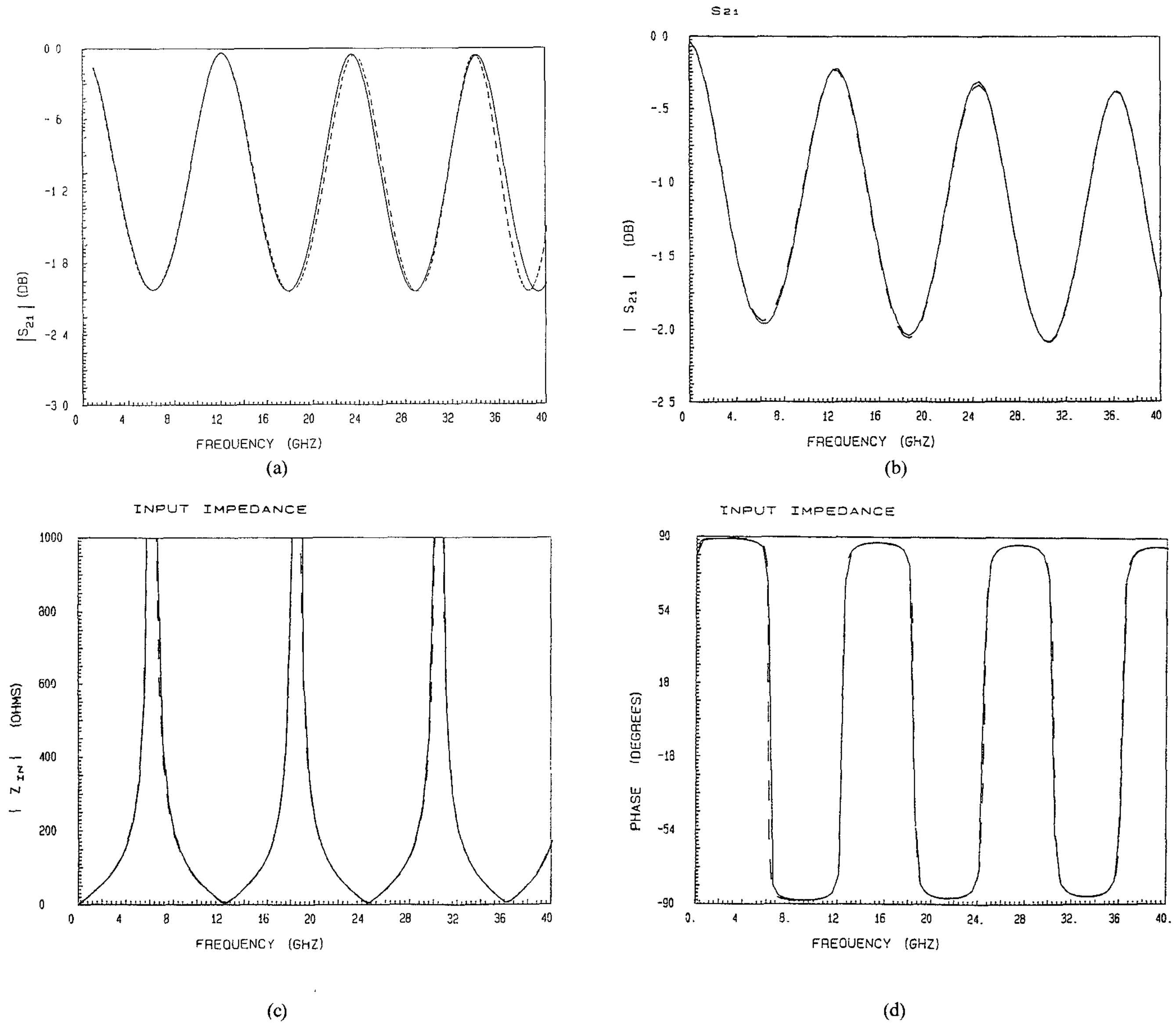


Fig. 4. (a) The $|S_{21}|$ of a 50-Ω mismatched microstrip. Termination 25 Ω. (b) The $|S_{21}|$ of the mismatched MMIC line. Termination 50 Ω. (c), (d) The amplitude and phase of the input impedance of MMIC open circuit stub. — Response of the model calculated with the CAD program SPICE; --- Computed by using the transmission line equations with complex line parameters.

closed-form empirical expressions for coupled lines derived from these [4]–[7]. Fig. 3 demonstrates the modeling accuracy for the nominal 50-Ω line on alumina. Here the attenuation constant, the normalized phase constant, and the characteristic impedance of line are compared with the same parameters calculated for the equivalent circuit model up to 40 GHz. As seen from these results, the losses and dispersion are modeled fairly accurately by the equivalent circuits and the modeling accuracy improves with the number of lumped element sections used to synthesize the augmenting series and shunt branches. Note that the first-order network representing the series branch consists of one RL and one LC section together with a series resistance, whereas a third-order section consists of three RL and three LC sections with a series resistance.

The transmission through the mismatched microstrip and the MMIC lines and the input impedance of the MMIC line stub are shown in Fig. 4. In both cases, 16 sections were used to model the lines. The 50-Ω microstrip on a 25 mil alumina substrate was 5 μm thick and 4.68 mm long, and the MMIC line was 10 μm wide, 5 μm thick, and 4.12 mm long and was deposited on a 7 mil SI GaAs substrate passivated by a 3 μm oxide layer. The first-order equivalent circuit model parameter values for the two cases were found to be (Fig. 1)

$$L_m = 0.22 \text{ mm}, \quad Z_0 = 50\ \Omega, \quad R_0 = 0.702\ \text{m}\Omega, \quad R_1 = 8.458\ \text{m}\Omega,$$

$$L_{11} = 0.127\ \text{pH}, \quad L_{21} = 32.73\ \text{pH}, \quad C_{21} = 0.114\ \text{pF}, \quad G_0 = 0.15\ \mu\text{S},$$

$$G_1 = 2.855\ \mu\text{S}, \quad L_{31} = 0.185\ \text{pH}, \quad C_{11} = 0.044\ \text{fF}, \quad C_{31} = 12.76\ \text{fF}$$

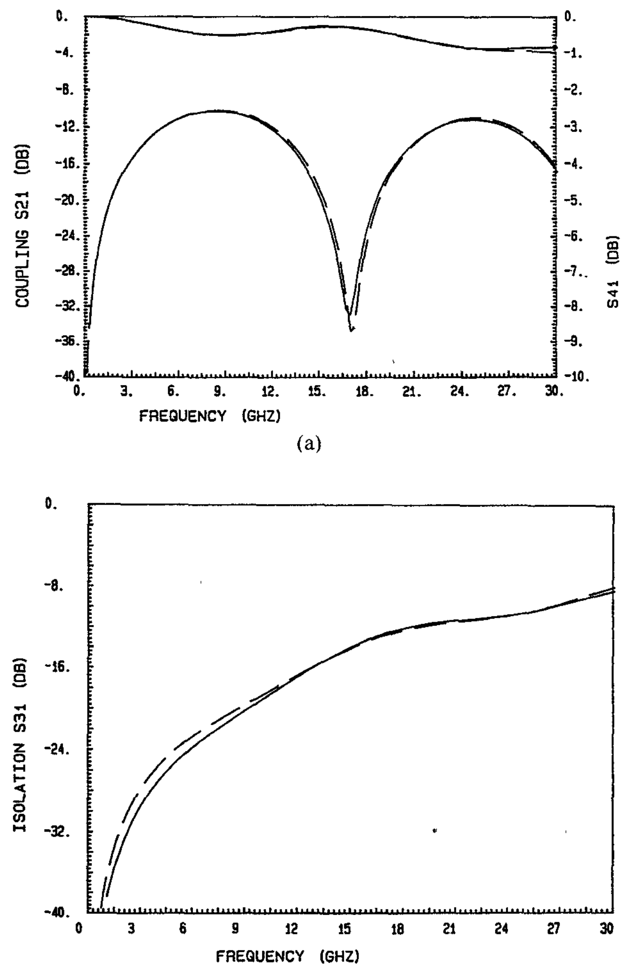


Fig. 5. The scattering parameters of an edge-coupled microstrip 10 dB coupler. 2 is the coupled port, 4 the direct port, and 3 the isolated port. 50 Ω terminations. — Response of the model calculated with SPICE; --- computed by using the complex S parameters of the four-port.

for the 50-Ω microstrip and

$$L_m = 0.2 \text{ mm}, \quad Z_0 = 98.5\ \Omega, \quad R_0 = 11.16\ \text{m}\Omega,$$
$$R_1 = 0.111\ \text{m}\Omega,$$
$$L_{11} = 1.45 \text{ pH}, \quad L_{21} = 29.79 \text{ pH}, \quad C_{21} = 82.49 \text{ fF},$$
$$G_0 = 1.15\ \mu\text{S},$$
$$G_1 = 10.23\ \mu\text{S}, \quad L_{31} = 0.543 \text{ pH}, \quad C_{11} = 0.141 \text{ fF},$$
$$C_{31} = 3.07 \text{ fF}$$

for the MMIC line.

The results obtained for the higher order resonance frequencies and the values of impedances and scattering parameters at higher frequencies obtained by using the circuit model on SPICE are in excellent agreement with the computed values (Fig. 3) and do demonstrate that the validity of the model in accurately representing the effects of dispersion and losses at higher frequencies. Note that only the lowest order circuits for Z and Y in the model were used and the agreement between the response of the model and the numerically computed values for the microstrips can be further improved by increasing the number of elements in these augmenting networks.

Fig. 5 shows the coupling, transmission, and isolation for a nominal 50-Ω, 10-dB edge-coupled microstrip coupler on alumina including dispersion and losses through 30 GHz. For this example, the length of the line = 3.4 mm, the microstrips are on 25-mil alumina with $W/H = 0.8$, $S/H = 0.3$, and $T = 5\ \mu$m. Both the even- and the odd-mode line length in the model are divided into 16 sections and each section was modeled in the same manner as a single lossy dispersive line section by using the first-order augmenting lumped element networks. The results calculated with SPICE by using the subcircuit models of Fig. 2(b) are shown in Fig. 5. The dashed curve represents the S parameters computed directly by using the four-port complex network functions (e.g., admittance matrix) of the lossy dispersive coupled line structure. It should be noted that if we were designing a mm-wave coupler the line length will be much smaller and the agreement between the results obtained from the model and those obtained by using the scattering matrix would be even better.

V. Concluding Remarks

In conclusion, a unified technique to develop equivalent circuit models for lossy dispersive single and coupled line structures has been presented. It is shown that starting from the numerically computed results or measured values of the frequency-dependent complex propagation constants and impedances for a given lossy dispersive single or coupled line structure, a circuit model consisting of RLC elements, ideal TEM lines, and linear dependent sources can in general be constructed whose frequency- and time-domain responses are the same as those of the actual lossy dispersive multiport. These equivalent circuit models consist of ideal lumped and distributed elements and hence are compatible with general computer-aided design techniques and programs. Both analog and digital circuits can be analyzed and designed with linear as well as active and nonlinear terminations by incorporating these models in CAD programs such as SPICE [12]. In addition, this technique provides an alternative useful approach to the empirical modeling of a host of useful structures for which accurate empirical closed-form solutions for proagation characteristics are not available. These include microstrips and coplanar waveguides on passivated substrates for MMIC's. The technique should be quite helpful in the computer-aided design of discrete and monolithic circuits and systems at microwave and millimeter-wave frequencies.

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